AD 684678

# FOREIGN TECHNOLOGY DIVISION

ON THE EQUATION OF RELATIVE WEIGHTS OF AN AIRCRAFT

by

N. A. Fomin

D D C

APR 8 1969

# EDITED MACHINE TRANSLATION

ON THE EQUATION OF RELATIVE WEIGHTS OF AN AIRCRAFT

By: N. A. Fomin

This document is a machine translation of Russian
text which has been processed by the AN/GSQ-16(XW-2)
Machine Translator, owned and operated by the United
States Air Force. The machine output has been post-
edited to correct for major ambiguities of meaning,
words missing from the machine's dictionary, and words
out of the context of meaning. The sentence word
order has been partially rearranged for readability.
The content of this translation does not indicate
editorial accuracy, nor does it indicate USAF approval
or disapproval of the material translated.

## DATA HANDLING PAGE

| 01-ACCESSION NO. | 98-DOCUMENT LOC | 39-TOPIC TAGS |
|---|---|---|
| TP8501702 | | aircraft industry, aircraft performance, civil aviation, flight mechanics, flight simulation, thrust to weight ratio |

| 09-TITLE ON THE EQUATION OF RELATIVE WEIGHTS OF AN AIRCRAFT |
|---|

| 47-SUBJECT AREA |
|---|
| 01 |

| 42-AUTHOR/CO-AUTHORS | 10-DATE OF INFO |
|---|---|
| FOMIN, N. A. | -----67 |

| 43-SOURCE IZVESTIYA VYSSHIKH UCHEBNYKH ZAVEDENIY. AVIATSIONNAYA TEKHNIKA          (RUSSIAN) | 68-DOCUMENT NO. FTD-MT-24-348-68 |
|---|---|
| | 69-PROJECT NO. 72302-78 |

| 63-SECURITY AND DOWNGRADING INFORMATION | 64-CONTROL MARKINGS | 97-HEADER CLASN |
|---|---|---|
| UNCL, 0 | NONE | UNCL |

| 76-REEL/FRAME NO. | 77-SUPERSEDES | 78-CHANGES | 40-GEOGRAPHICAL AREA | NO. OF PAGES |
|---|---|---|---|---|
| 1886   1326 | | | UR | 9 |

| CONTRACT NO. | X REF ACC. NO. | PUBLISHING DATE | TYPE PRODUCT | REVISION FREQ |
|---|---|---|---|---|
| | 65-AP8002029 | 94-00 | Translation | None |

| STEP NO. | ACCESSION NO. |
|---|---|
| 02-UR/0147/67/000/004/0149/0154 | |

**ABSTRACT**

Some applications of the relative weights of an aircraft and the relative weight coefficients are investigated. The entire weight (or mass) $G_c$ of an aircraft is composed of the terms

$$G_c = G_k + G_{su} + G_t + G_{sng}.$$

The four G terms on the right-hand side of this equation denote, respectively, the structural weight of the aircraft, the weight of all power equipment, the weight of fuel, and the weight of the crew, equipment, and payload. Dividing both sides of the total weight equation by $G_c$ yields four relative weight coefficient terms whose sum is unity. These coefficients are useful in determining, among other factors, certain performance and economy characteristics of cruising speeds. The author concludes that the equation of relative weights is the basic equation which establishes the relationships between the flying characteristics of an aircraft and its parameters. Orig.art. has: 18 formulas and 2 figures.

BLANK PAGE

## U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|-------|--------|-----------------|-------|--------|-----------------|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

\* ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.
The use of diacritical marks is preferred, but such marks
may be omitted when expediency dictates.

# FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

## DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

| Russian | English |
|---------|---------|
| sin | sin |
| cos | cos |
| tg | tan |
| ctg | cot |
| sec | sec |
| cosec | csc |
| sh | sinh |
| ch | cosh |
| th | tanh |
| cth | coth |
| sch | sech |
| csch | csch |
| arc sin | $\sin^{-1}$ |
| arc cos | $\cos^{-1}$ |
| arc tg | $\tan^{-1}$ |
| arc ctg | $\cot^{-1}$ |
| arc sec | $\sec^{-1}$ |
| arc cosec | $\csc^{-1}$ |
| arc sh | $\sinh^{-1}$ |
| arc ch | $\cosh^{-1}$ |
| arc th | $\tanh^{-1}$ |
| arc cth | $\coth^{-1}$ |
| arc sch | $\text{sech}^{-1}$ |
| arc csch | $\text{csch}^{-1}$ |

———

| Russian | English |
|---------|---------|
| rot | curl |
| lg | log |

# ON THE EQUATION OF RELATIVE WEIGHTS OF AN AIRCRAFT

## N. A. Fomin

The total weight (mass) of an aircraft is composed of several parts with distinguishing features:

$$G_c = G_\kappa + G_{cy} + G_\tau + G_{c\vartheta r}, \tag{1}$$

where $G_\kappa$, $G_{cy}$, $G_\tau$, and $G_{c\vartheta r}$ are, respectively, the structural weight of the aircraft, the weight of all power equipment, the weight of fuel, and the weight of systems, equipment, crew, and payload.

The $G_\kappa$ depends on a number of parameters of the aircraft and its parts, mainly on wing loading p, wing aspect ratio $\lambda$, coefficient of rated overload $n_\lambda$, the weight of the aircraft, etc.

The $G_{cy}$ depends on the weight of the engine, the value of thrust of the aircraft, the weight of tanks, etc.

The $G_\tau$ depends on specific fuel consumption, range of the aircraft, its cruising speed, weight of the aircraft, etc.

The $G_{c\vartheta r}$ in general is not directly connected with the parameters and characteristics of the aircraft or its weight and is determined in accordance with technical requirements, depending upon the type of aircraft and its function.

If we divide both sides of (1) by $G_c$ we will obtain the equality

$$1 = \overline{k}_{\kappa} + \overline{k}_{cy} + \overline{k}_{\tau} + \overline{k}_{c\theta r},\tag{2}$$

to which attention was turned for the first time by V. F. Bolkhovitdinov [1]; it is called the equation of relative weights of an aircraft, where $\overline{k}_{\kappa} = \dfrac{G_{\kappa}}{G_{c}}$, $\overline{k}_{cy} = \dfrac{G_{cy}}{G_{c}}$, $\overline{k}_{\tau} = \dfrac{G_{\tau}}{G_{c}}$, $\overline{k}_{c\theta r} = \dfrac{G_{c\theta r}}{G_{c}}$ are the relative weights of the structure, propulsion system, fuel, equipment, crew, and loads. The relative weight of the propulsion system can be presented as

$$\overline{k}_{cy} = \overline{t}_0 \cdot r_0,\tag{3}$$

where $\overline{t}_0$ is the starting trust-to-weight ratio of an aircraft with a turbojet engine and $r_0$ is the specific weight of propulsion system.

The relative weight of the fuel load $\overline{k}_T$ can be expressed as follows: for a given duration t' of flight of an aircraft with turbojet engine:

$$\overline{k}_{\tau} = s \# \overline{t}_0 C_{\tau_0} \Delta t',\tag{4}$$

and for a given range $L_{max}$ of flight of aircraft with a turbojet engine at $M_{\text{кр}}$ = const and $H_{\text{кр}}$ = var,

$$\overline{k}_{\tau} = s R \# C_{\tau_0} \frac{L_{max}\sqrt{c_{x_0} D_0}}{M_{\text{кр}}} + u,\tag{5}$$

where s = 1.15-1.20 — a coefficient considering aeronautical margin,
R = 0.00145, if $\overline{k}_T \leq 0.3$,
R = 0.001, if $0.3 < \overline{k}_T < 0.5$,
u = 0.0009 $H_{\text{кр}}$, if $\overline{k}_T \leq 0.3$ and M < 2.0,
u = 0,0009 $H_{\text{кр}} + \psi \xi c_{T_0} \overline{t}_0 t''$, if $0.3 < \overline{k}_T < 0.5$ and M > 2.0,
u = 0.0009 $H_{\text{кр}} + 0.09$, if $0.3 < \overline{k}_T < 0.5$ and M < 2.0,

$\psi$ — coefficient, considering the influence of speed of flight on specific fuel consumption,

$\xi$ — coefficient considering the influence of speed of flight on engine thrust,

$\Delta$ — relative air density,

$t'$ — time of flight (for Formula 4),

$t''$ — time of acceleration [takeoff run] (for Formulas 4 and 5),

$C_{TO}$ — specific fuel consumption,

$c_{x_0}$ — drag coefficient at $c_y = 0$, $D_0 = c_{x_1}/c_y^2$

$M_{\text{кр}}$ — Mach number corresponding to cruising speed,

$H_{\text{кр}}$ — mean value of cruising altitude.

Using the equation of relative weights (2) and the given equalities (3)-(5) it is possible to obtain expressions for "available" thrust-to-weight ratio $\bar{t}_{0p}$ (turbojet engine):

$$\bar{t}_{0p} = \frac{1 - \bar{k}_{\text{к}} - \bar{k}_{\text{сит}}}{\bar{r}_0 + s\psi\xi C_{T_0}\Delta t'}, \tag{6}$$

if duration of flight, is given, and

$$\bar{t}_{0p} = \frac{1 - \bar{k}_{\text{к}} - \bar{k}_{\text{сит}}}{\bar{r}_0} - \frac{sR\psi C_{T_0} L_{\text{max}}\sqrt{c_{x_0}D_0}}{\bar{r}_0 M_{\text{кр}}} - \frac{u}{\bar{r}_0}, \tag{7}$$

if range is given; they permit judging about the character of the influence of one or another parameter on the "available" thrust-to-weight ratio of the aircraft. It is necessary to keep in mind that use of the formula

$$\bar{t}_0 = \frac{P_{0\text{су}}}{G_{\text{с}}},$$

is meaningful only for designed aircraft; to find $\bar{t}_0$ for projected aircraft in this way is impossible, since $G_{\text{с}}$ is a quantity dependent of $P_{0\text{су}}$. Analysis of (6) and (7) permits judging about the qualitative influence of different parameters on $\bar{t}_{0p}$.

Quantitatively this influence can be determined with help of the formulas

$$\bar{t}_0' = \bar{t}_0 \frac{1 - \varepsilon_1 \bar{k}_{\kappa}}{1 - \bar{k}_{\kappa \dots}};$$

$$\bar{t}_0' = \bar{t}_0 \frac{1 - \varepsilon_2 \bar{k}_{\tau}}{1 - \bar{k}_{\tau}};$$

$$\bar{t}_0' = \bar{t}_0 \frac{1 - \varepsilon_3 \bar{k}_{\tau}}{1 - \bar{k}_{\tau}}; \tag{8}$$

$$\bar{t}_0' = \bar{t}_0 \frac{1}{1 + \bar{k}_{c\gamma}(\varepsilon_4 - 1)};$$

where $\varepsilon_1 = \frac{\bar{k}_{\kappa}'}{\bar{k}_{\kappa}}$, $\varepsilon_2 = \frac{C_{\tau_0}'}{C_{\tau_0}}$, $\varepsilon_3 = \frac{L_{max}'}{L_{max}}$, $\varepsilon_4 = \frac{r_0'}{r_0}$; $\bar{t}_0'$ is a new value of $\bar{t}_0$, obtained if $\bar{k}_{\Pi}$ is increased by $\Delta\bar{k}_{\Pi}$. If we increase thrust-to-weight ratio $\bar{t}_0$, installing on the aircraft two engines instead of one, at $L_{max}$ = const, $G_{c\ni\Gamma}$ = const and $\bar{k}_{\kappa}'$ = $\varepsilon_1\bar{k}_{\kappa}$:

$$\bar{t}_0' = \frac{1 - (\bar{k}_{\kappa}\varepsilon_1 + \bar{k}_{\tau})}{\frac{0.5\bar{k}_{c\ni\Gamma}}{\bar{t}_0} + r_0\varepsilon_4}.$$

The formulas in (8) can easily be obtained from the equation of relative weights:

$$\bar{k}_1 + \bar{k}_2 + \bar{k}_3 + \bar{k}_4 = 1.$$

If $\bar{k}_1$ is increased by $\Delta\bar{k}_1$, then $\bar{k}_2$, $\bar{k}_3$, and $\bar{k}_4$ will be changed and the equation will take the form

$$\bar{k}_1 + \Delta\bar{k}_1 + \bar{k}_2' + \bar{k}_3' + \bar{k}_4' = 1.$$

Then $\bar{k}_i' = \bar{k}_n \left(1 - \frac{\Delta\bar{k}_1}{1 - \bar{k}_1}\right)$, $\Delta\bar{k}_n = -\bar{k}_n \frac{\Delta\bar{k}_1}{1 - \bar{k}_1}$, $G_i' = G_i \frac{1}{1 - \frac{\Delta\bar{k}_1}{1 - \bar{k}_1}}$.

4

Setting $P'_{cy} = P_{cy}$, we will obtain

$$\bar{t}_0' = \bar{t}_0\left(1 - \frac{\Delta\bar{h}_1}{1 - \bar{h}_1}\right)$$

or

$$\bar{t}_0' = \bar{t}_0\left(1 - \frac{\Delta\bar{h}_{\text{н}}}{1 - \bar{h}_{\text{н}}}\right) \qquad (9)$$

For actual realization of an aircraft with a prescribed M number of horizontal flight to be possible requires first of all equality of "available" and "required" thrust-to-weight ratio. The latter can be expressed on the basis of the equalities

$$M_{\text{max}} = 0{,}0148\sqrt{\frac{\bar{p}\bar{t}_0\ell}{c_x}}$$

and

$$M_{\text{кр}} = 0{,}012\sqrt{\frac{\bar{p}\bar{t}_0\ell}{c_{x_0}}}.$$

Thus,

$$\bar{t}_{0\text{п}} = \frac{4650\,M_{\text{max}}^2\,c_x}{\ell p}, \text{ if } M_{\text{max}} \text{ is given,}$$

or

$$\bar{t}_{0\text{п}} = \frac{6060\,M_{\text{кр}}^2\,c_{x_0}}{\ell p}, \text{ if } M_{\text{кр}} \text{ is given.} \qquad (10)$$

Equating $\bar{t}_{0p}$ and $\bar{t}_{0\text{п}}$, we have

$$\frac{1 - \bar{k}_{\text{н}} - \bar{k}_{\text{сbr}}}{r_0 + s\psi^{t\Delta}C_{T_0}\ell'} = \frac{4650\,M_{\text{max}}^2\,c_x}{\ell p} \qquad (11)$$

5

and

$$\frac{1-\overline{k}_{u}-\overline{k}_{cor}}{r_0} - \frac{sR\Psi C_{y_0} L_{max} V\overline{c_{x_0} D_0}}{r_0 M_{sp}} - \frac{u}{r_0} = \frac{6950 M_{sp}^2 c_{x_0}}{\xi p}. \qquad (11')$$

From this we will obtain equations with respect to M which enabling us to determine that value of $M_{max}$ or $M_{кp}$ which is possibly and practically realizable for an aircraft with the selected parameters:

$$\overline{k}_{u}, \ \overline{k}_{cor}, \ p, \ c_{x_0}, \ C_{y_0}, \ r_0, \ L_{max} \ \text{or} \ \ell'.$$

$$4650 M_{max}^2 c_x (r_0 + s\Psi \underline{\Delta} C_{y_0} \ell') - \xi p (1 - \overline{k}_u - \overline{k}_{cor}) = 0, \qquad (12)$$

$$6950 M_{sp}^2 c_{x_0} - M_{sp} \xi p (1 - \overline{k}_u - \overline{k}_{cor} - u) + sR\Psi C_{y_0} L_{max} p \sqrt{c_{x_0} D_j} = 0. \qquad (12')$$

Any greater value of M can be found with decrease

$$L_{max}, \ \overline{k}_u, \ r_0, \ C_{y_0}, \ c_{x_0}, \ \ell'.$$

Any smaller value of M will provide the possibility of increasing

$$L_{max}, \ \overline{k}_{cor}, \ \ell'.$$

Solving the first equation (12), we will obtain for M at a given duration t'

$$M_{max} = \sqrt{\frac{\xi p (1 - \overline{k}_u - \overline{k}_{cor})}{46500 c_x (r_0 + s\Psi C_{y_0} \Delta\ell')}}$$

$$(13)$$

Note. For determination of the value of $M_{max}$ it is possible to use graphic procedure. Pursuing several values M, determine $\xi$, $c_x$,

6

and $\psi$ and by (13) calculate $M_{max}$. Construct a curve in coordinates $M_{зад}$ and $M_{ист}$ (Fig. 1); $M_{max}$ can be found from its intersection with a straight line drawn from the origin of the coordinates at an angle of $45°$; $M_{зад}$ and $M_{ист}$ must be taken in the same scale.
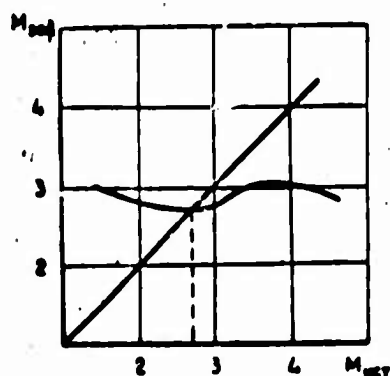


Fig. 1. Example of graphic solution of equation (13).

The quantity $M_{кр}$ can be found also, using a graphic solution (Fig. 2), taking the following functions for plotting of curves:

$$6950\, M_{кр}^{2}\, c_{x_0}\, r_0 = \theta_1$$

and

$$M_{кр}\, \xi\, p\, (1 - \bar{k}_s - \bar{k}_{сог} - u) -$$
$$- sR\gamma C_{b}\, L_{max}\, \xi\, p\, \sqrt{c_{x_0}}\, D_{q} = \theta_2 \qquad (14)$$
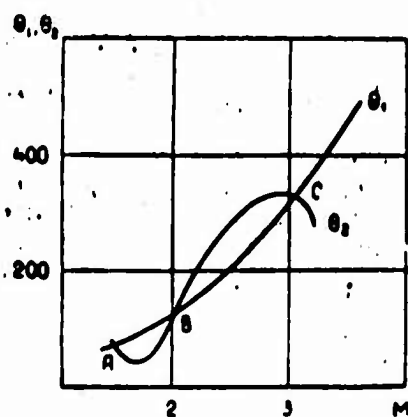


Fig. 2. Example of graphical solution of equation (14). In points A, B, and C a given value of $L_{max}$ can be obtained, other things being equal and with corresponding values of $M_{кр}$ and $t_0$. Between points A and B the given value of $L_{max}$ and other things being equal cannot be obtained.

The point of intersection of these curves on the graph in coordinates M, $\theta_1$, and $\theta_2$ will determine the unknown value $M_{\text{кp}}$.

From (13) and (14) it is easy to see that to each M, at constant values of all parameters, there will correspond its own $\overline{k_{\text{к}}}$, which can be called the "necessary" specific weight of the structure of the aircraft. Using (11) and (11'), it can be expressed:

$$\overline{k}_{\text{к}_\text{s}} = 1 - \overline{k}_{\text{cor}} - \frac{4650 \, M_{\text{м03}}^2 \, c_x}{t \rho} \, (r_0 + s \psi \Delta \, C_{y_0} \, l') \tag{15}$$

or

$$\overline{k}_{\text{к}_\text{s}} = 1 - \overline{k}_{\text{cor}} - \frac{6950 \, M_{\text{кp}}^2 \, c_{x_0} \, r_0}{t \rho} - \frac{s R \psi \, C_{y_0} \, L_{\text{м03}} \sqrt{c_{x_0} \, D_0}}{M_{\text{кp}}} \,. \tag{16}$$

The "available specific weight of the structure $\overline{k}_{\text{кp}}$ is the weight which can be obtained at given strength of the structure and selected parameters of wing, fuselage, etc. It can be expressed by these formulas:

for aircraft with sweptback wing,

$$\overline{k}_{\text{кp}} = \left( 0.027 \, \psi \, n_A \frac{G_c^{\frac{1}{2}}}{\cos \chi} \sqrt{\frac{\lambda}{\rho} + \frac{5.5}{\rho}} \right) (1 + \beta_1 \lambda_\phi \, m + \beta_2) + 0.065, \tag{17}$$

for aircraft with delta wing of small aspect ratio,

$$\overline{k}_{\text{кp}} = \left( 0.049 \, \psi \, n_A \, G_c^{1/2} \sqrt{\frac{\lambda}{\mu} + \frac{5.5}{\rho}} \right) (1 + \beta_1 \lambda_\phi \, m + \beta_2) + 0.065. \tag{18}$$

Here $G_c$ – in m total weight (mass) of aircraft,

$\beta_1 = 0.07$-$0.09$ – for supersonic aircraft,

$\beta_1 = 0.065$-$0.08$ – for heavy subsonic and transonic aircraft,

$\beta_1 = 0.08$-$0.115$ – for transonic transport aircraft,

$$m = 1 - \text{for supersonic aircraft,}$$

$$n = 1.2\text{--}1.3 - \text{for subsonic and transonic aircraft,}$$

$$\beta_2 = 0.27 - \text{for supersonic aircraft,}$$

$$\beta_2 = 0.15 - \text{for subsonic and transonic aircraft,}$$

$$\varphi = 1 - \frac{3(\eta + 1)}{\eta + 2}(\bar{z}_1 \epsilon_1 \bar{k}_\tau + \bar{z}_2 \epsilon_2 \bar{k}_{cy}) - \text{coefficient of relief of wing,}$$

$\eta$ — wing taper,

$\epsilon_1$ — portion of fuel in wing,

$\bar{z}_1$ — relative coordinate in fractions of semispan of CG of fuel in wing,

$\bar{\epsilon}_2$ — portion of weight of propulsion system in wing,

$\bar{z}_2$ — relative coordinate in fractions of semispan of CG of power plant.

The major problem of designing reduced to ensuring the equality of "available" $\bar{k}_{кр}$ to "necessary" $\bar{k}_{к_п}$.

On the basis of the material outlined above it can be concluded that the equation of relative weights of an aircraft is the fundamental equation with which the relationships between the flight characteristics of an aircraft and its parameters are established.

## Literature

1. Bolkhovitdinov V. F. Puti razvitiya letatel'nykh apparatov (Paths of development of aircraft). Oborongiz, 1962.

2. Fomin N. A. Proyektirovaniye samoletov (Designing of aircraft). Oborongiz, 1961.

3. Badyagin A. A., Cvrutskiy Ye. A. Proyektirovaniye passazhirskikh samoletov s uchetom ekonomiki ekspluatatsii (Designing of passenger aircraft, taking into account economics of operation). Mashinostroyeniye, 1964.